Emphasizing the Identity by Integrating the Cultural Dimension in the Design Aiding Tools
"Paradigm on the Design Tools of Islamic Geometric Patterns"

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Abstract
The aiding tools for design started as early as art creation and have evolved over time since then. These tools played a very important role to mediate between theory and practice in design; It also aims to facilitate the artisan's work to create sophisticated art that usually conveys messages and meanings drawn from a specific culture. Tracking the evolution of the design aiding tools would help to establish a generic model that would facilitate tools development through a clear methodology.
The Islamic geometric patterns are proper paradigm to use in reaching this development as it constitutes a mathematical challenge that needs aiding tools to simplify the process of producing such art.
Several studies have been conducted in order to investigate the construction techniques of Islamic patterns; even though, those studies focused on the mathematical analysis of the patterns form rather than the applied construction techniques used by the artisans, which are believed to be varied according to cultural background. The Islamic geometric patterns found to have common features that make such pattern easily recognizable; even though, they have variations resulted from cultural background for each region. Therefore, the compensation is needed between **commonality and variation** in the same time. The cultural variation is the absent dimension in methods and tools used to create Geometric Islamic pattern. By integrating **what is missing** (culture dimension in our case) into the aiding tools and affords it to designers and artisans will help in creating significant art with **specific identity**.

Keywords:
Islamic Geometric Pattern; From Theory to Practices in Design; Mediating Tools Aiding design; Cultural DNA; Shape Grammar.

Introduction
The aiding tools for design started as early as art creation and have evolved over time since then. These tools played a very important role to mediate between theory and practice in design; and also facilitate the artisan's work to create sophisticated art that usually conveys messages and meanings drawn from a specific culture.
Accordingly, the evolution of aiding (mediating) tools varying from design domain to another and noticeably artisans (and recently designers) are relying on those tools more and more gradually. Therefore, studying the capability and range of aiding tools in a specific design domain will provide better understanding and ease of handling any emerging design dimensions over time. A selective design domain would serve as a paradigm to conduct the required

DOI:. 10.21608/mjaf.2020.43739.1866

investigation towards intended goal, and similarly the method will be applicable for other design domains such as: architecture, interior design, and all forms of art. The Islamic geometric patterns are proper paradigm to use in reaching this development as it constitutes a mathematical challenge that certainly needs aiding tools to simplify the process of producing such art.

Research Problem:

Applying the concept in hand; it has been found that several studies have been conducted in order to investigate the construction techniques of Islamic patterns; even though, those studies focused on the mathematical analysis of the patterns form rather than the final production of the art by the artisans, which are believed to be varied according to cultural background. The Islamic geometric patterns have common features that make such pattern easily recognizable; even though, they have variations resulted from cultural background for each region. This dimension is not integrated so far in aiding (mediating) tools.

Research Significance:

As long as the cultural dimension is absent in methods and tools used to create Geometric Islamic pattern the final product will be recognized in common (Islamic Pattern) but will be incomplete regarding culture and place. The compensation is needed between **commonality and variation** in the same time. By integrating **what is missing** (culture dimension in our case) into the aiding tools and affords it to designers and artisans will help in creating significant art with **specific identity**.

Research Objective:

Achieving a comprehensive understanding of the evolution of aiding tools of the Islamic geometric patterns to detect what is covered and what is missing. Then integrate cultural features that are accountable for variations (different identities) within the commonality of a specific art (Islamic geometric patterns in this case).

Research Methodology:

The research will follow the **inductive approach** through historical tracking for the aiding design methods (tools) and then **analytical approach** to determine what is missing in those methods and then proposing a model to understand and guide the aiding tools evolution:

- Defining the design domain (emergence factors of Islamic geometric patterns).
- Tracking the evolution of the Islamic geometric patterns' aiding tools (construction methods) reaching the advanced computational ones.
- Analyzing the affecting factors of the Islamic geometric patterns that resulting commonality versus variation in such art.
- Suggesting the integration of the cultural dimension into relevant design aiding tools to help in producing art identity.

Definition of Islamic geometric patterns and their emergence factors

1. Islamic Art and Faith

Islamic art has been recognized around the world through ages as an example of the Muslim artisan's brilliance. Muslim artisans have shown remarkable creativity by representing their beliefs using series of abstract forms and shapes while rejecting, in general, the depiction of living forms. Art work for Muslims has been a way of conveying the message of Islam besides being an artistic product. This explains why a remarkable attention has been given to geometric pattern by scientists, mathematicians, and artisans.

1. Like other aspects of Muslim culture, Muslims' art was a result of the accumulated knowledge of local environments and societies, incorporating Arabic, Persian and African traditions, in addition to Byzantine inspirations [1] Muslims built on this knowledge and developed their own unique style, inspired by **three main factors**:

a. **The Qur'an** is seen as the first work of art in Islam. If we go through the verses of Qur'an we can notice the independence of some and interrelation of others that form astonishing meanings. Each verse takes the reader into a unique divine experience varying from joy, happiness, and bliss to fearfulness and alertness to calmness and satisfaction. [2]

Al-Faruqi, R. (1973) stated: "The constant repetition of these experiences in the verses of Qur'an winds up consciousness and generates in it a momentum which launches it on a continuation or repetition and infinitum" [2]

Muslim artists used the lessons learned from Qur'an to develop a new approach of art characterized by the independence and interdependence of its formative elements.

b. Connecting the work with the **remembrance of God**. This formula regulated the approach of Muslim artists and writers, who used various materials and methods in expressing their remembrance.

c. Dictating the nature of Muslims' art is the religious rule that **discourages the depiction of human or animal form.** [1] This rule was due to the concern that people would go back to worship idols and figures. Consequently, **Islamic beliefs** are considered to be the main factor governing Islamic art. These beliefs were sustained and passed on from one generation to the next by acceptance rather than just copying which explains the pattern survival and evolution.[3] Islamic art was mainly classified into three main types (excluding the figurative art): floral, geometrical and calligraphy.

2. Islamic Art and Culture

The artisans as members of community interact positively by absorbing the surrounding cultural features and reflecting back messages and enhancing the quality of the society through artwork. As we might expect, within this span of time and space many variations and styles occur; yet, Islamic art remains somehow "recognizable" throughout time. Commonly, the focus of Islamic art was towards "applicable art" such as: Architecture, the arts of the book (calligraphy, manuscript illumination, and bookbinding), and the arts of the object (especially metalwork, ceramics, glass, and textiles). Figure (1)

FIGURE (1). (A) LAMP (EGYPT OR SYRIA 1345) – (B) CHILD'S KAFTAN WITH TIGER-STRIPE DESIGN (TURKEY, 16TH CENTURY) – (C) MINBAR FOR SULTAN QA'ITBAY CAIRO, (1468 AND 1496), AND (D) DETAIL [4]

Many **cultural factors** have direct effect that shaped the Islamic art identity; one significant factor is **Muslims' Belief:** As illustrated in Figure (1-a) The lamp with non-figural decoration that inscribed with verses from the Light Sura (24.35) in The Holy Quran: "God is the light of the heavens and the earth; the likeness of his light is as a niche wherein a lamp, the lamp in a glass, the glass as it were a glittering star". That is a proof of the direct impact of the belief on the Muslim's artwork.

Also, the Child's kaftan with tiger-stripe design (Figure 1-b) recall the tiger-skin robe worn by the Persian mythological hero Rustam. That identifies another variable which is the **effect of literature-folk stories on the artwork**.

The variable in the interest is clearly appears in the Minbar for Sultan Qa'itbay and its details, (Figure 1-c, and 1-d) Sixteen-rayed stars and spokes define a variety of polygons. The star is based on a four-part division of a circle: imagine a square rotating within it. The elaboration of geometrical designs reached a peak in the fourteenth century, in Egypt, Syria, Iran, and Spain. It is clear in this one the **effect of sophisticated mathematics on the artwork.** [4]

The previous examples clearly demonstrate the direct effect of culture on the produced art; therefore, the cultural context is clearly influences the process of generating the art and appears in the art main features; that would be a variation within common practice of Islamic art.

Initially, an important factor regarding Islamic geometric pattern emergence is aiding methods and tools that facilitate producing such sophisticated art.

3. Islamic Art and Science (Mathematics)

Mathematics recognized as advanced science in the Muslim world due to its essential use in a variety of practical operations related to Islamic practices. For example, times of prayer, divisions of inheritance, and direction of Qibla (to Makkah) in addition to other branches of math such as: trigonometry, spherical geometry, algebra and arithmetic that were the base of accurate religious observances. [1]

Mathematical sophistication in the Islamic world led to the evolution of the geometric pattern; accordingly, the applied art was developed as well through aiding tools and techniques to ease such complex applications. Figure (2)

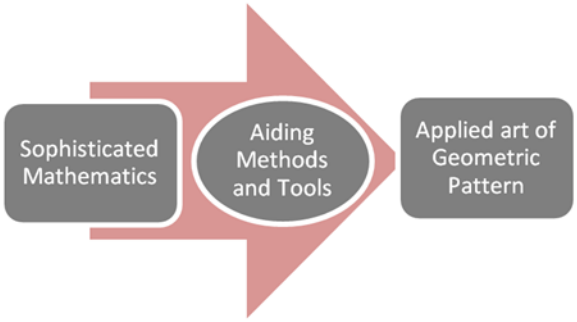

Figure (2): Aiding methods and tools facilitate geometric pattern creation

After presenting the three main components of the Islamic geometric patterns emergence (belief, culture, and Mathematics); in the next part of the research, we will go through the evolution of aiding tools of the pattern construction methods. This part is meant to be as a point of interest in this research. Then; a model for better understanding to make the aiding tools more specific to generate art with identity which will be suggested.

Tracking the evolution of the Islamic geometric patterns' aiding tools (construction methods)

Background: Mathematics and construction methods

Throughout history many theories were introduced to help in the development of mankind and it was the role of engineers/artisans to convert these theories into application. The process of converting theory to application usually does not change, what changes is the way of conversion. In Islamic pattern what converted the pattern from geometrical theory into aesthetic application on objects and building facades is the construction method.

The purpose of this analysis is to investigate aiding tools through Islamic history and explore how to provide contemporary artisan (designer) with new methods to facilitate the creation of genuine art and architecture as a continuation of the on-going process that form the logical evolution of such aiding tools. Also, detect any missing factors that would help assuring the **identity** of the resulted art.

Continues communication between mathematicians and artisans

Many methods were introduced as attempts of developing construction methods based on new acquired knowledge and techniques. Accordingly, continuous interaction between mathematicians and artisan would lead to better output regarding Islamic geometric patterns. Figure (3) describes the flow of knowledge back and forth to insure accuracy of the final result of applied geometric patterns. Tracking the evolution of such process will lead to inference that supports the research goal.

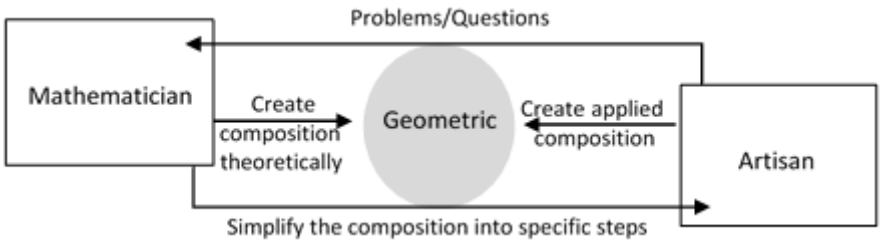

Figure 3: Interaction between mathematicians and artisans in process of creating geometric pattern

Tracking the aiding tools (construction methods) over the time

Upcoming chapter is presenting a sample of continuous aiding tools (construction methods) regarding Islamic geometric pattern over time; as following up the evolution of aiding tools would establish the significant role of such tools and expected advancement in the same flow.

1. Notes of Abu Al-Wafa Al-Buzjani

Abu Al-Wafa Al-Buzjani (940–998 A.D.) tells us in his book, "On the Geometric Constructions Necessary for the Artisan" (Kitab fıma yahtaju ilayhi al-sani min al-a mal al-handasiya); hereinafter Geometric Constructions, that he attended meetings between mathematicians and artisans in Baghdad. Such meetings were a widespread phenomenon in the Islamic world in the 11th century in Isfahan. [5]

The notes taken by Abu Al-Wafa in Geometric Constructions and the descriptions in Interlocks of figures are a very rare documentation of the meeting between mathematicians and artisans. Abu Al-Wafa represented two different methods of pattern construction to simplify the process which are: **a)** Verging Method, and **b)** Cut and Paste Method. [6]

a) Verging Method

In the verging method, approximation by means of (T-square) is used to achieve the desired pattern. The following are pages from Abu Al-Wafa's book in which the verging method is explained (Figure 4).

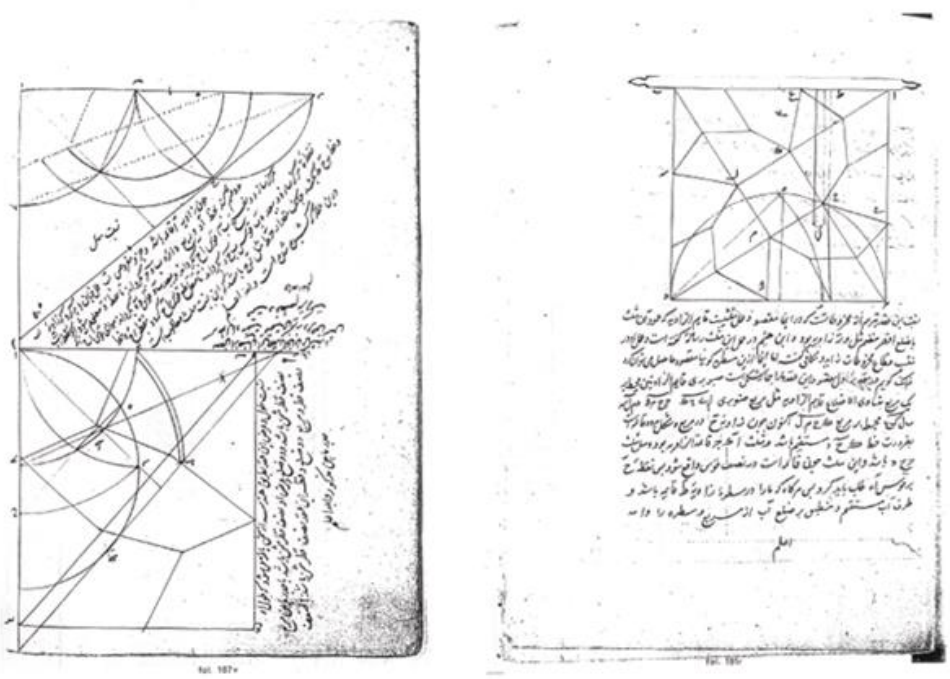

FIGURE (4): PAGES FROM ABU AL-WAFA AL-BUZJANI'S BOOK [5, 6]

From the figure above, the first page shows the mathematical analysis of the pattern, whereas the other page contains a set of instruction for the artisan in order to draw the pattern using a tool (T-square). In verging method, Islamic pattern is achieved by approximation and cutting down the problem into a set of steps, which can be easily followed to reach the ultimate goal (Figure 5).

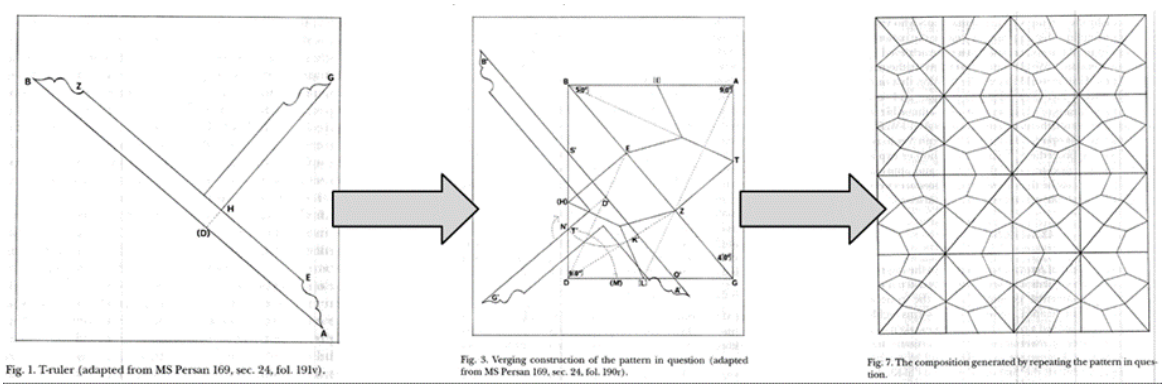

FIGURE (5): ILLUSTRATION OF THE TOOL USED BY MUSLIM ARTISANS IN VERGING METHOD AS SHOWN IN ABU AL-WAFA'S BOOK [6] (ARRANGED BY THE AUTHOR).

Since the patterns were made using preliminary tools, each additional step means an additional possible source of inaccuracy in geometric constructions in general.

b) Cut and Paste Method

In the method of Cut and paste the technique of cutting and putting shapes together is followed to achieve the desired results. It is believed that this method was developed by artisans who were skillful in inlaying. Carpenters fit this description very well, whose job involved cutting a single piece of material into parts and arranging those parts in patterns composed of different figures (Figure 8).

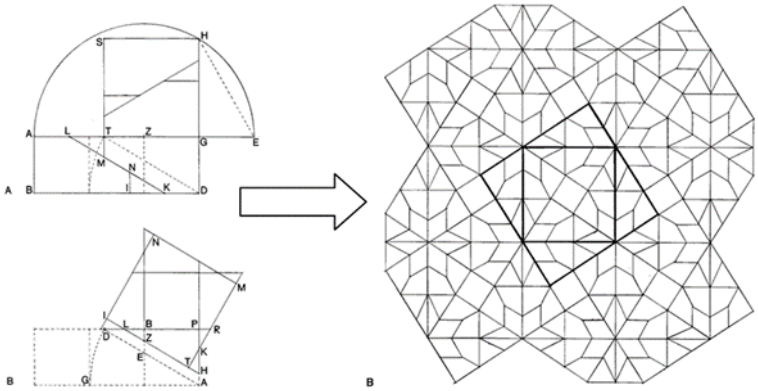

FIGURE (6): ILLUSTRATION OF CUT AND PASTE METHOD [7]

In the hands of creative artisans, this method turned out to be very effective and lead to the discovery of a number of new patterns that became quite popular. This cooperation between mathematicians and artisans enriched Islamic geometric pattern and made it a unique form of art.

2. Strap-work – Geometry based construction

In strap-work circles and squares are transformed into stars and overlapping lattice to form an intricate symmetric pattern (Periodic method). In this method an overlying geometry is made, which acts as guidelines, following which artisans can make the pattern. This method is similar to graph paper. Figure (7)

A lot of mathematical analyses were conducted on the strap-work method, trying to categorize it and finding common features between different patterns. From various analyses of this method, it can be concluded that it was highly **flexible and complicated** at the same time. One type of grid can end up with numerous patterns, making it difficult to trace all.

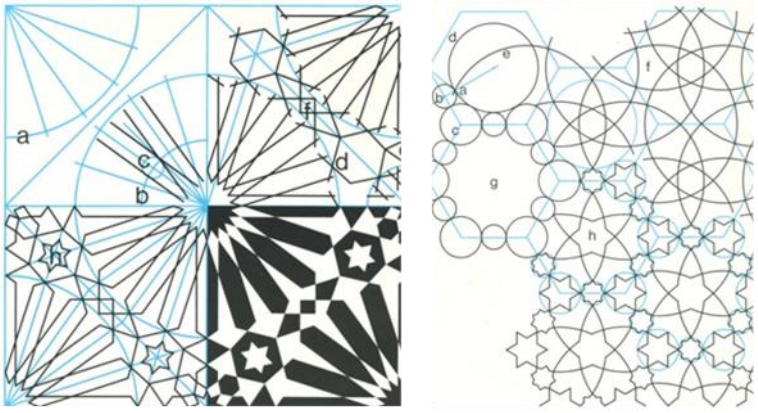

FIGURE 7: DAVID WADE'S ANALYSIS OF ISLAMIC PATTERN [8]

David Wade's analysis of the pattern is on individual basis; these analyses are very complicated and cannot be considered as a tool used by artisans for constructing the pattern (except in some simple cases). This method is complicated and general guidelines couldn't make it an easy way to be implemented, so it fails to be an efficient tool for artisans even though it is considered as a great analytical method for constructing variation of pattern based on underlying geometry.

3. Tilling based construction – Girih Tilling [9]

Girih tilling is a form of sophisticated tilling composition which gives a quazi-crystaline pattern composition. Old method found in Topkapi scroll in Topkapi Palace Museum [9] decoded in 2006, by Peter Lu [10]. It is classified as Quasi-crystalline pattern which means it doesn't expand in symmetrical way in both directions as the regular Islamic pattern (Crystalline). The Girih pattern can expand in any direction without losing its regularity. The remarkable use of Girih tilling in Islamic pattern design emphasizes the importance of orderly system. Because Girih represents the complete world, free of defects and misleading notions, as a world of wisdom where all its parts and the whole are arranged in a predetermined order. [10]

"The Girih represents a disciplined consistent world, in which each Girih represents everything measured and evaluated, and has a place of its own. So the choice and placement of every detail is inevitable and could not be otherwise" (Lu, Peter, 2007)

Girih tilling can be considered as flexible tools, which allowed artisans some freedom in expressing their creativity in the art work and maintaining the genuine characteristics of the Islamic pattern (Figure 8).

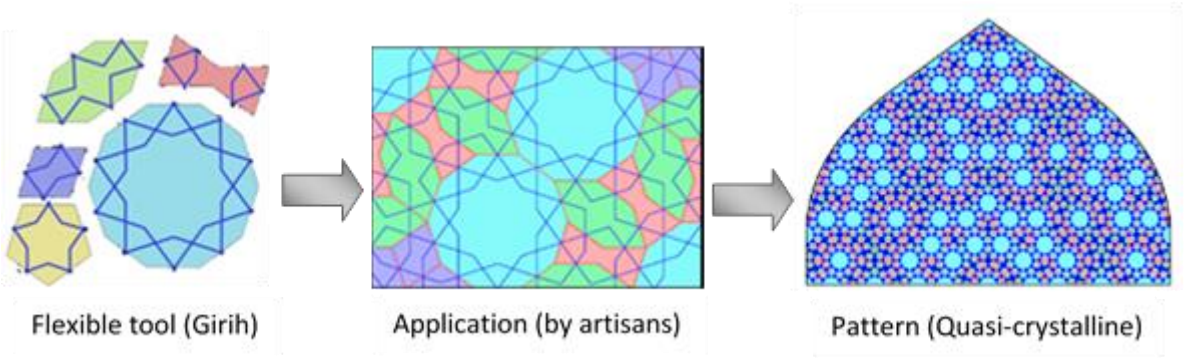

Figure (8): Elements of Girih tiles and their use in constructing pattern [10]

4. Hankin's Polygon-in-Contact Method [11]

Hankin came up with this method in the early twentieth century. According to him, to make such patterns, first cover the surface with a network consisting of **polygons in contact**, then through the center of each side of each polygon two lines are drawn. These lines cross each other like a letter X and are continued till they meet other lines of similar origin. Figure (9)

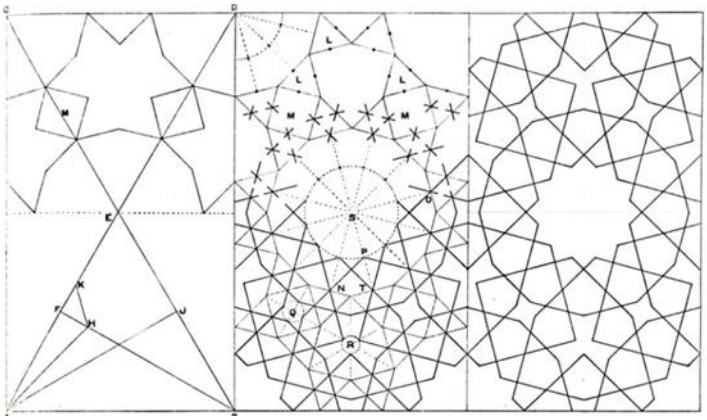

Figure (9): Hankin's Polygon in Contact method [11]

This method although being clear in theory was difficult to apply, which made it inconvenient to be used by artisans. This method remained theory until Kaplan employed its principal into computer application (2002).

Later Jay Bonner, an architectural ornamentalist in New Mexico, worked on classifying Islamic star pattern into categories according to different angles based on Hankin's Polygon in Contact method (Figure 10) [12].

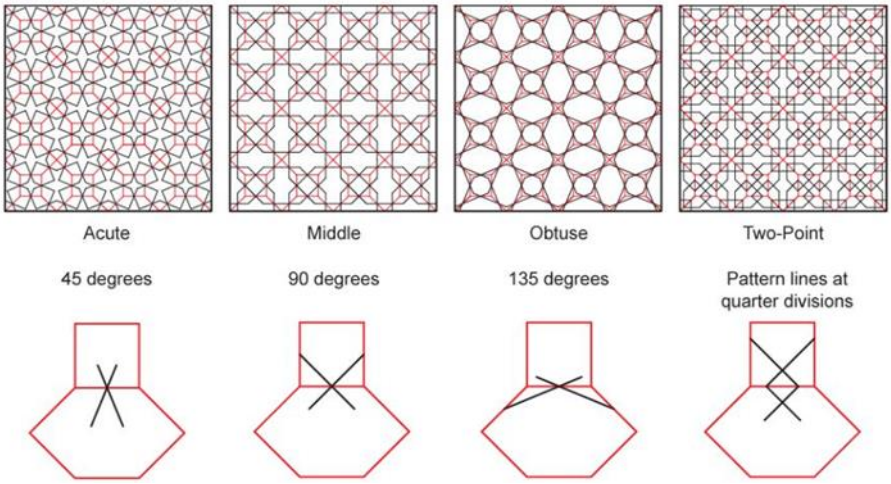

Figure (10): Bonner's categorization of Islamic star pattern [12]

The evolution in application tools haven't stopped yet, fascination with the complexity of Islamic pattern has led to development of new computer-based application tools that would help in generating variety of patterns.

5. Kaplan's computational application of Hankin's method

As mentioned earlier, Hankin's method remained theory until Kaplan converted it into a software application.[13] What made Kaplan succeed in this conversion is changing the tool that suits the method, earlier Hankin did not have any possibility for a medium other than manual tools, but because of the technological advancement this conversion was possible.

Hankin's method gave some freedom in generating patterns using computer program. Kaplan (2002) explored Hankin's method and developed a Java program that implemented this method. At its core the program is an "inference algorithm": a subroutine that takes a polygonal tile and a contact angle as input and returns a motif for that tile. The inference algorithm runs quickly enough that the contact angle can be varied in real time, with the changes to the resulting design displayed interactively, Figure (11). [13]

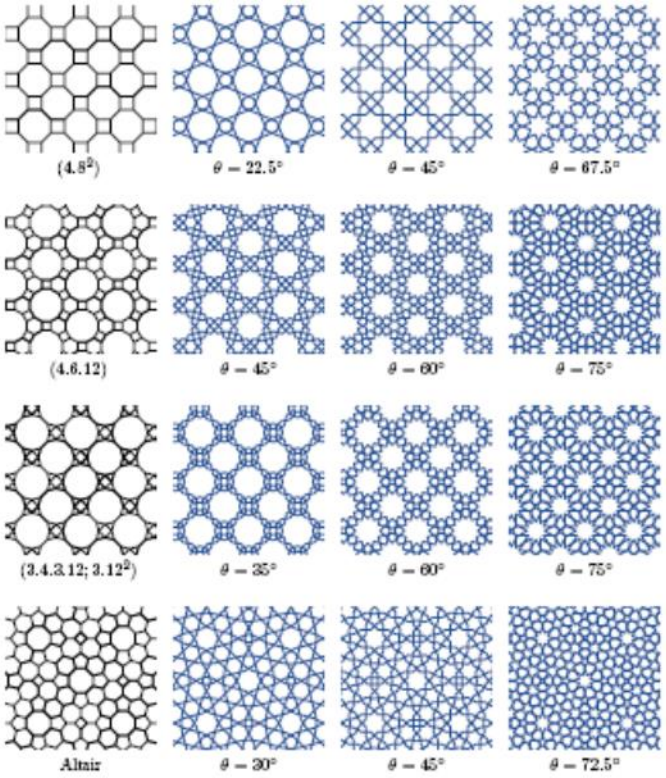

Figure (11): Hankin's method of construction implemented on computer application by Kaplan [13]

Over the years, many techniques have been proposed, and all are successful in various ways. The wide variety of successful techniques reflects the improbability that there was ever a single historical design method. More likely, the artisan's toolkit held an **assortment of mathematical.** Any technique that can create a large variety of well-known patterns can be judged as success. Modern mathematics might even reveal degrees of freedom in pattern construction that were unavailable in the past because **the tools to understand them had not yet been developed.** [13]

Kaplan's software has been a step forward in employing state of the art technologies that enable us to explore new methods.

6. Employing "Shape Grammar" in generating Islamic geometric pattern [14]

In this research authors applied a relatively new tool in computer applications that aids design methods; "Shape Grammar" to facilitate generating Islamic geometric pattern. The application was limited to **Moorish style** pattern as a beginning.

A shape grammar consists of shape rules that define geometric properties of a particular style. The rules can then be applied to shapes to generate existing or new instances of the style. For the purposes of this study, a corpus of eight Islamic geometric patterns was analyzed. Each of the patterns in the corpus exhibited a repeating square pattern with an eight-fold rotational symmetry, and the underlying structure could be described according to a triangular region, the contents of which serve as the motif of the pattern. The resultant shape grammar is illustrated in figure (12).

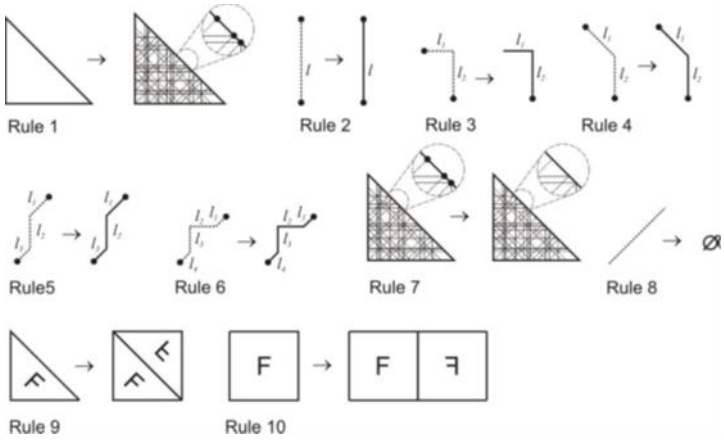

Figure (12): Shapes rules of the Islamic geometric pattern grammar [14]

Applying rules in different ways will ensure the emergence of the intended pattern with Moorish style. **Here was the first step towards a specific result representing a definite identity**.
As the application of a shape grammar is initialized by a **seed shape**, which for the Islamic geometric pattern grammar is a **right-angled triangle**, as illustrated in figure (13). The applied rules are moving from one design to the next as specified. [14]

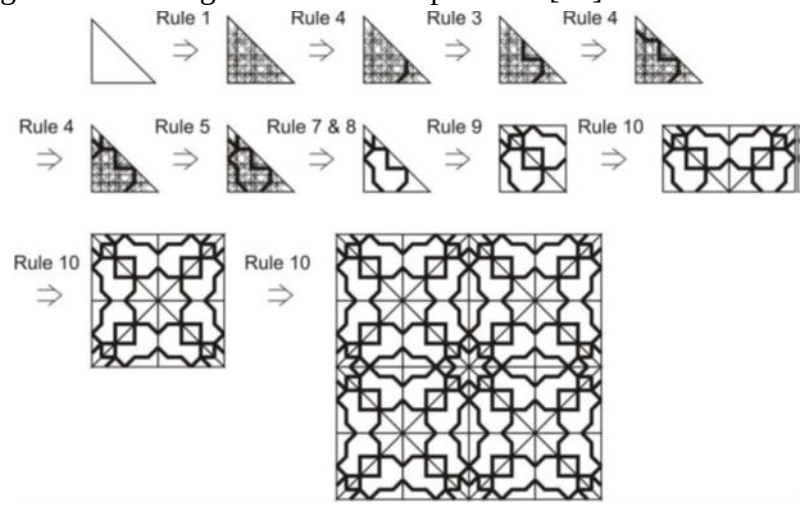

Figure (13): Motif-based construction of an Islamic geometric pattern. [14]

Results and Discussion

Analyzing the affecting factors of the Islamic geometric patterns (Commonality vs. Variation)

The methods analyzed so far are simply techniques for helping artisans solve their "problem" of making Islamic pattern more easily and efficiently. In order to create a contemporary aiding tool, further points should be highlighted. **Generalization**, which Tony Lee mentioned in his research, is a way of simplifying the information and is also a step towards creating a tool, which would help in generating products with characteristics of the Islamic art. [15] Nevertheless, the **variation** is present in Islamic geometric patterns due to the varied cultural background for artisans and art creators.
Therefore, it can be concluded that the Islamic geometric patterns have three main factors affecting the resulted art product: 1) Belief, 2) Culture, and 3) Science (Mathematics).

1) Belief: is a fundamental factor that directly **leads to commonality**. That is why anywhere we can recognize the geometric patterns as Islamic art. They are all connected to the main concept and main source.
2) Culture: is an inherited factor responsible for **variation** within the general concept of Islamic geometric patterns.
3) Science (Mathematics): is evolving and resulting **sophistication** that eventually needs more capable aiding tools to bear the complexity in the process.
Accordingly, understanding these affecting factors is considered one **main finding** of this research as explained in figure (14);

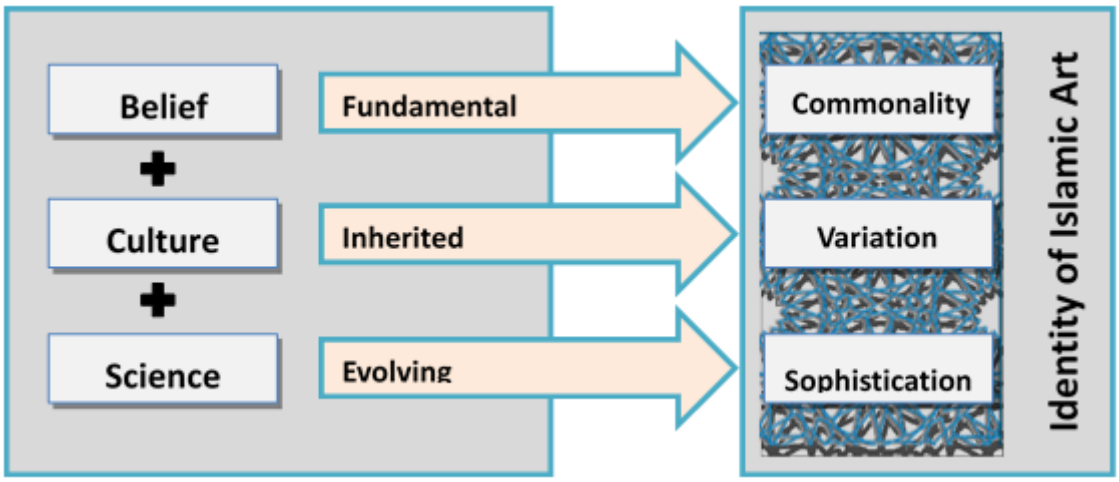

FIGURE (14): ILLUSTRATING THE FACTORS THAT COMPOSE THE IDENTITY OF ISLAMIC ART

Islamic pattern has the feature of variety and commonality, through which it can be easily recognized in any part of the world and preserves the individuality of styles in the same time; that magnitude need to be facilitated by new aiding tools.
For example, according to Reki and Selçuk (2018) Their study examines how the patterns evolved over the time and how the great Mughals, who were originally from Central Asia, descended from the ethnic Turks of Mongol, adopted these patterns in the Indian subcontinents art and architecture. [16]
Therefore; once "**materials and colors"** governed by artisans' local environment; those patterns will have their own cultural identity. Hypothetically, the aiding techniques can be considered as **expert systems** (as knowledge containers) once built cannot remain static, but must be adaptable to include new knowledge as it becomes available and accumulates over time. The new knowledge to be integrated into this expert system (aiding tools) would be **the cultural aspects** which usually are revealed by using specific **materials and colors** that express each culture. By having a look at figure (15) we can easily recognize the following: 1) All pictures represent Islamic geometric pattern (easily recognized owing to commonality from Belief), 2) We can recognize, more or less, the region of cultural context affected the patterns' materials and colors as well, and 3) Sophistication depends on the time of creation of the pattern.

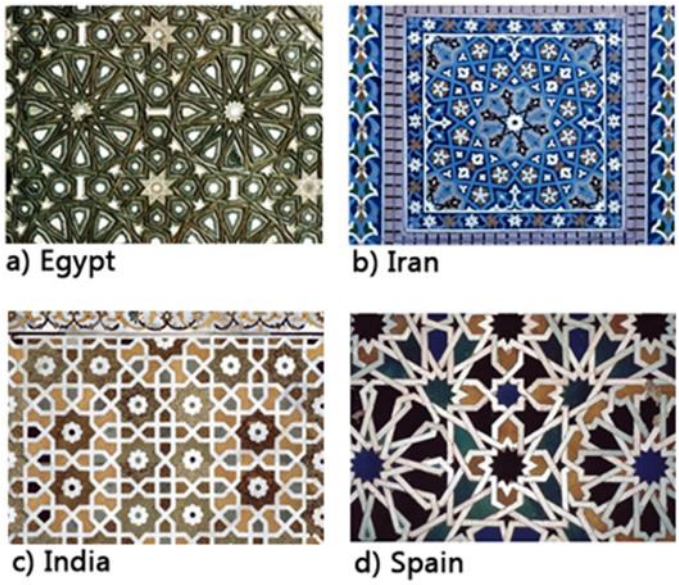

FIGURE 15: COMMONALITY: ALL ARE ISLAMIC GEOMETRIC PATTERNS, SOPHISTICATION REGARDING THE EVOLUTION OF MATHEMATICS AND AIDING TOOLS AS WELL, AND VARIATION REGARDING THE CULTURE OF THE REGION OF ORIGIN

Integrating the cultural dimension into relevant design aiding tools model

By considering the inherited culture into aiding tools that will allow the designer to create sophisticated patterns and assure **specific identity in the same time**. Figure (16)

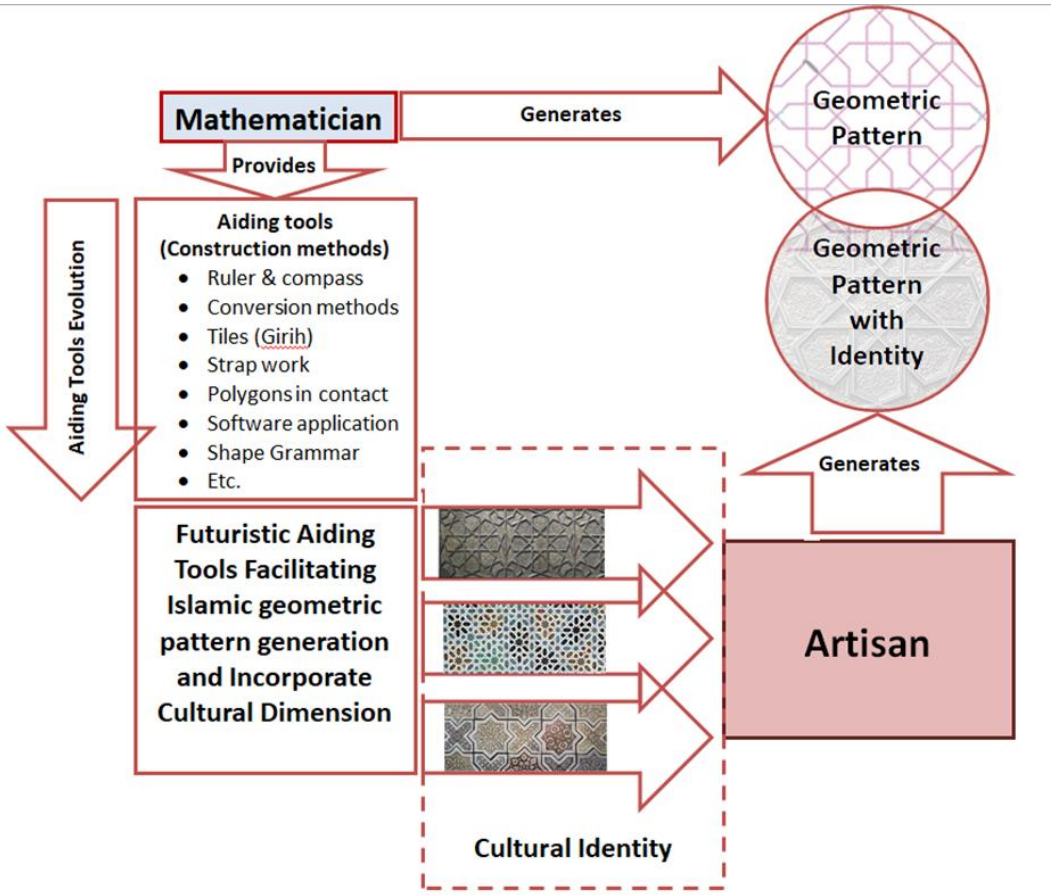

Figure (16): Figure illustrating the importance of providing artisans with proper tools and techniques incorporate cultural dimension to produce art with identity

According to the suggested model in figure (16); the new computational tools would be supplied with a library that has different culture attributes regarding material and colors to select from according to the wanted style. That step will be next to the creation of the required pattern using the construction methods employed by the tool itself. A more advanced way is by utilizing "cultural DNA"; that will be stated in future work.

Conclusion

Recently, artisans, and designers as well, are depending on advanced aiding tools to facilitate their artwork production. The cultural dimension is very essential to insure the artwork identity. Therefore, integrating the culture dimension into the aiding tools will help the artisan in a way to generate art with specific identity without ending up with undesired eclectic forms. The suggested model introduced an approach to develop aiding tools for the artisan (and designers) with the option to integrate cultural style during art creation process. That would be possible, for example, by choosing out of attached digital library that include colors, materials, and other attributes related to each culture.

Finally what applies for Islamic geometric pattern can be generalized to all other design domains that require specific style and form related to specific culture.

Future work

Introducing the concept of cultural DNA is a promising approach to insure generating patterns (or designs in general) with a specific identity. In 1976, Dawkins identified cultural "memes" as part of the cultural DNA, a term used for the elements that are units of cultural selection. [3, 17]

"The memes as units of cultural inheritance sustain the survival and replication of cultural environment, however, if they have no effect the content deteriorates and disappears with time. Thus as DNA in biology, the memes should have cultural information to be preserved and maintained throughout generations". [3]

The upcoming research approach will be in creating a tool that would reflect the Muslim culture identity that replicates the cultural DNA. These memes, once identified, will serve as the common feature for the suggested aiding tools.

Therefore, future suggested research is to provide the contemporary designer (artisan) with an aiding tool that would facilitate creating patterns (or any design related to culture and style) that would reflect the identity and carry the cultural memes, which makes it easily recognizable, while maintaining the different styles imposed by different regions.

References:

[1] Saoud. R. (2004). Introduction to Muslim Art. Foundation of Science, Technology and civilisation, UK, FSTC Limited. 4-6

[2] Al-Faruqi, R. (1973). Islam and Art, Studia Islamica, Vol 37, Larose, Paris, pp. 81-110.

[3] Distin, K. (2005). The Selfish Meme. Cambridge University Press, New York, USA.

[4] National Gallery of Art, (2004) Islamic art and culture - a resource for teachers. Board of Trustees, National Gallery of Art, Washington.

[5] Abul-Wafa al-Buzjani. (940-998). Kitab Fi Ma Yahtaju Ilayh Al-Sani' Min Al-A'mal Al-Handaseya: Imam Riza 37, Baghdad (Persian manuscript)
[6] Özdural, A. (1996). On Interlocking Similar or Corresponding Figures and Ornamental Patterns of Cubic Equations. In Muqarnas Volume XIII: An Annual on the Visual Culture of the Islamic World. Gülru Necipoglu (ed.). Leiden: E.J. Brill.
https://archnet.org/publications/4465 (accessed: November 2019)
[7] Özdural, A. (2000). Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World, Historia Mathematica 27, 171–201.
https://www.sciencedirect.com/science/article/pii/S0315086099922747 (accessed: January 2020)
[8] Wade, Dd. (1976). Pattern in Islamic Art. The Overlook Press, the university of Michigan
[9] Necipoglu, G. (1956). The topkapi scroll-geometry and ornament in Islamic architecture. Topkapi Palace Museum Library MS H, Published by The Getty Center for the History of Art and the Humanities, USA, 1995.
[10] Lu, Peter J. & Steinhardt, Paul J., (2007). Decagonal and Quasi-Crystalline Tiling in Medieval Islamic Architecture. Science, Vol. 315, Issue 5815, pp. 1106-1110. https://science.sciencemag.org/content/315/5815/1106.full (Accessed: March 2020)
[11] Hankin, E. H. (1925). The drawing of geometric patterns in Saracen art. Government of India Central Publication Branch (1998 Edition)
[12] Bonner, Jay (2003). Three traditions of self-similarity in fourteenth and fifteenth century Islamic geometric ornament. ISAMA/ Bridges 2003 Proceedings, pp. 1-12.
[13] Kaplan. C. S. (2002). Computer graphics and geometric ornamental design. PhD Diss. University of Washington.
[14] Jowers, I., Prats, M., Eissa, H., & Lee, J. (2010). A Study of Emergence in the Generation of Islamic Geometric Patterns. Proceedings of the 15th International Conference on Computer Aided Architectural Design Research in Asia CAADRIA, 39-48
[15] Lee .A. J. (1975). Islamic star patterns – notes. summaries of main results of researches into the geometry of Islamic star patterns during Nov. 1964.
[16] Reki, M. & Selçuk, S. A. (2018). Evolution of geometric patterns in Islamic world and a case on the Jalis of the Naulakha pavilion in the Lahore fort. Journal of Science, Gazi University, GU J Sci, Part B, 6(1): 83-97
[17] Distin, K. (2012). Cultural Evolution. Cambridge University Press.